WOBURN CHALLENGE

2016-17 Online Round 1

Sunday, October 16th, 2016

Senior Division Problems

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Problem S1: Hide and Seek

15 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: wcipeg.com/problem/wc161s1*

It's Halloween night in Haddonfield, Illinois. Not exactly in the mood for trick-or-treating, Laurie Strode has instead invited her friend, Michael Myers, to join her for a friendly game of hide and seek.

Laurie's house has a hallway with a number of rooms. The floor plan of this section of the house can be represented as a string with N ($1 \le N \le 200{,}000$) characters, each of which is either a "`.`" (representing empty space) or a "`#`" (representing a wall). A room is a maximal consecutive sequence of empty space in this floor plan. That is, each room is either preceded by a wall or is at the start of the floor plan. Similarly, each room is followed by a wall or is at the end of the floor plan. The floor plan includes at least one room.

No doubt you're familiar with the rules of hide and seek – Laurie is hiding in one of the rooms, and it's Michael's job to find her. When he enters a room, he can immediately determine if Laurie is hiding anywhere in it, so he could simply visit each of the rooms in the hallway and be sure to find her eventually.

However, maybe he can do even better than that. Michael has a superb sense of hearing, so when he enters a room, he can also determine if Laurie is hiding in any of the rooms which are no further than D ($1 \le D \le 10^9$) units away from that room. The distance between two rooms (in units) is equal to the minimum distance between any parts of those rooms in the floor plan (in characters). For example, if the floor plan is "`.#..##..`", then the distance between the left room and the middle room is 2, the distance between the middle room and the right room is 3, and the distance between the left room and the right room is 6.

Michael is certainly looking forward to winning this game of hide and seek, but he values efficiency. He's wondering – what's the minimum number rooms which he must enter in order to guarantee that he'll determine Laurie's location, no matter which room she's hiding in? In particular, to ensure victory, every room in the hallway must be within D units of at least one of Michael's chosen rooms.

In test cases worth 3/15 of the points, $N \le 1000$ and $D = 1$.
In test cases worth another 6/15 of the points, $N \le 1000$.

Input Format

The first line of input consists of two space-separated integers N and D. The second line consists of a single string representing the floor plan.

Output Format

Output one line consisting of a single integer – the minimum number of rooms which Michael must enter to determine Laurie's location.

Sample Input

```
22 4
..#.#...##.#.....###.#
```

Sample Output

```
2
```

Sample Explanation

There are 6 rooms in the house. If Michael enters the 3rd room from the left, he'll be able to hear if Laurie is in any of the 4 leftmost rooms. If he enters the 5th room from the left, he'll be able to hear if she's in any of the 3 rightmost rooms. Therefore, entering the 3rd and 5th rooms from the left will be sufficient. Entering the 3rd and 6th rooms from the left would also do the job.

Problem S2: Alucard's Quest

20 Points / Time Limit: 4.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc161s2*

What a horrible night to have a curse! Alucard has returned to the ancient castle of his evil father, Dracula, determined to wake him from his slumber and then destroy him once and for all. However, something tells him that it won't be easy - though Dracula remains fast asleep in his coffin for now, his monstrous servants are scattered throughout the castle, armed to the teeth and hungry for blood.

The castle consists of N ($1 \le N \le 200{,}000$) chambers, with $N - 1$ passageways running between them. The i-th passageway connects distinct chambers A_i and B_i ($1 \le A_i, B_i \le N$), and has M_i ($1 \le M_i \le 5000$) monsters in it. It's possible to reach any chamber from any other chamber by following a sequence of one or more passageways – in other words, the system of chambers and passageways forms a tree structure when modelled as a graph.

Dracula's resting place is in the 1st chamber, and fortunately for Alucard, he's already infiltrated the castle and also finds himself in the 1st chamber! However, he's realized that he's not quite ready to battle Dracula yet. In order to stand a chance, Alucard will surely need some holy water, stronger weapons (a whip should come in handy), a wider range of magical spells to cast, and of course an oak stake to plunge into his father's heart and finish him off permanently. In particular, Alucard will first need to gather K ($1 \le K < N$) items. Conveniently, all of these items can be found in distinct chambers of Dracula's castle, with the i-th item in chamber C_i ($2 \le C_i \le N$).

Alucard will need to travel around the castle through its passageways, starting from the 1st chamber, visiting all K chambers that contain his required items (in any order), and arriving back in the 1st chamber to finally wake and confront Dracula. If he chooses to travel through a passageway that contains m monsters, he'll first need to destroy them by casting a spell and using up m of his "magic points". That passageway will then be permanently cleared of monsters, so he'll be able to freely travel through it any number of times afterwards.

Conserving magic points for his battle with Dracula is vital, so Alucard will need to carefully plan out a route through the castle which will allow him to collect all K items while requiring him to use as few magic points as possible. Can you help him?

In test cases worth 3/20 of the points, $N \le 1000$ and $K = 1$.
In test cases worth another 7/20 of the points, $N \le 1000$.

Input Format

The first line of input consists of two space-separated integers N and K.
$N - 1$ lines follow, with the i-th of these lines consisting of three space-separated integers A_i, B_i, and M_i (for $i = 1..N - 1$).
K lines follow, with the i-th of these lines consisting of a single integer C_i (for $i = 1..K$).

Output Format

Output one line consisting of a single integer – the minimum number of magic points required for Alucard to collect all K items and return to Dracula's chamber.

Sample Input

```
7 4
1 2 5
1 7 2
2 4 3
2 5 8
5 6 1
7 3 10
4
5
3
7
```

Sample Output

```
28
```

Sample Explanation

One optimal route that Alucard can take, passing through all 4 chambers that contain items and then returning to the 1st chamber, is as follows:

- 1 → 7 (2 magic points)
- 7 → 3 (10 magic points)
- 3 → 7 (already cleared)
- 7 → 1 (already cleared)
- 1 → 2 (5 magic points)
- 2 → 4 (3 magic points)
- 4 → 2 (already cleared)
- 2 → 5 (8 magic points)
- 5 → 2 (already cleared)
- 2 → 1 (already cleared)

The total number of magic points required on this route is 2 + 10 + 5 + 3 + 8 = 28.

Problem S3: Tricky's Treats

30 Points / Time Limit: 2.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc161s3*

Oh boy! It's Tricky's favourite time of year, Halloween! He can't wait to put on his awesome dinosaur costume and take a walk around his neighbourhood. Of course, what he's looking forward to the most are the loads of treats his neighbours are sure to give him!

Tricky lives at one end of a long street. There are N ($1 \le N \le 10^5$) other houses on the street, with the i-th one being P_i ($1 \le P_i \le 10^9$) metres away from his own house. No two houses are at the same location. If Tricky visits the i-th house and scares its residents with a fearsome dinosaur roar, he's sure to receive C_i ($1 \le C_i \le 10^4$) treats from them. Of course, he can only visit each house at most once, though.

Tricky would love to get as many treats as possible, but one thing may stand in his way - time! He absolutely must be home no later than midnight (that's when the real monsters come out). Fortunately, he may be getting quite a head start on his trick-or-treating expedition - he'll be leaving home M ($1 \le M \le 43{,}200{,}000$) milliseconds before midnight.

Despite his elaborate costume and the potential weight of many treats, Tricky can get around pretty quickly. He can walk down the street in either direction at a rate of 1 metre per millisecond. If he decides to stop at a house along the way to collect its treats, it'll take him T ($1 \le T \le 10^4$) milliseconds to do so before he can continue on his way.

Given that Tricky must be back in the safety of his house at the end of the street after no more than M milliseconds of trick-or-treating, how many treats can he possibly end up with, and what's the probability of him developing long-lasting health issues as a result of the following excessive sugar consumption? (Just kidding, he'll be just fine!)

In test cases worth 3/30 of the points, $N \le 10$ and $M \le 2000$.
In test cases worth another 6/30 of the points, $N \le 500$ and $M \le 2000$.
In test cases worth another 6/30 of the points, $N \le 1000$.

Input Format

The first line of input consists of three space-separated integers N, M, and T. N lines follow, with the i-th of these lines consisting of two space-separated integers P_i and C_i (for $i = 1..N$).

Output Format

Output one line consisting of a single integer – the maximum number of treats that you can receive while still returning home by midnight.

Sample Input

```
4 2000 500
123 4
400 20
100 5
751 999
```

Sample Output

```
25
```

Sample Explanation

One optimal strategy is for Tricky to walk to the 2nd house, collect its treats, walk to the 3rd house, collect its treats, and then walk back home. This takes 400 + 500 + 300 + 500 + 100 = 1800 milliseconds, and yields a bounty of 20 + 5 = 25 treats. If the 4th house were 1 metre closer to Tricky's house, then he would have just enough time to walk straight to it, collect its 999 treats, and make it back home at midnight.

Problem S4: TP

35 Points / Time Limit: 2.00s / Memory Limit: 256M *Submit online: wcipeg.com/problem/wc161s4*

You and your friends have had enough of this Halloween nonsense. You nabbed your fair share of candy in the past, sure, but you wouldn't be caught dead parading around in a costume now that you're in high school. That doesn't mean this dumb excuse for a holiday has to go to waste, though... No way. It's time to have some real fun.

There are N ($2 \le N \le 10^7$) nice-looking houses on your street, but they won't be looking nice for long. Having borrowed your family car for the night, you'll be taking a joy ride down the street with your friends and some bathroom supplies in tow. Your plan is to take M ($1 \le M \le 2000$) passes along the street, and on each pass, you'll choose a random pair of distinct houses (chosen uniformly at random from all possible pairs) to be your targets. Then, those two houses will each receive a little punishment... Bam! Toilet paper to the face!

Once you've TP-ed a house in this fashion, it will be thoroughly covered in the stuff. It may get chosen as a target again on future passes, but applying toilet paper multiple times will have no additional effect on it.

This is sure to be a sweet night, but now you're wondering just how much carnage you and your friends will inflict on your lame neighbourhood. What's the expected number of different houses which will have been TP-ed at least once by the end of Halloween?

In test cases worth 4/35 of the points, $N \le 5$ and $M \le 5$.
In test cases worth another 5/35 of the points, $N \le 10$ and $M \le 10$.
In test cases worth another 18/35 of the points, $N \le 1000$ and $M \le 1000$.

Input Format

The first and only line of input consists of two space-separated integers N and M.

Output Format

Output one line consisting of a single real number – the expected number of houses which will be covered in toilet paper after all M passes, with at most 10^{-5} absolute or relative error.

Sample Input

```
3 2
```

Sample Output

```
2.666667
```

Sample Explanation

There are 3 possible pairs of distinct houses – 1&2, 1&3, and 2&3. Therefore, there are 9 equally-likely pairs of pairs of houses which will be TP-ed on the two passes. 3 of these result in only 2 distinct houses being covered in toilet paper (for example, 1&2 being targeted on both passes). The other 6 of them result in all 3 houses being covered. Therefore, the expected number of houses which will be covered in toilet paper is (3×2 + 6×3)/9 = 8/3.