WOBURN CHALLENGE

2017-18 On-Site Finals

Sunday, May 13th, 2018

Junior Division Problems

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Problem J1: Cownterintelligence

10 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: http://wcipeg.com/problem/wc17fj1*

The Great Cow-Monkey War of 2017 has left Scarberia in a fragile and unstable state. Interspecies tension is at an all-time high, and both sides are working relentlessly to rebuild their economies, social systems, and militias. Little do the cows and monkeys know that, amidst the turmoil, the world is at risk of being taken over by an even more dangerous foe — a group of aliens known as the Party of Extraterrestrial Gangsters (or PEG for short)! PEG has decided that it's the perfect time to divide and conquer Scarberia by exploiting its current internal hostilities.

Prior to their invasion, PEG would like to obtain some intel on the military of each group, starting with the cows. The entire cow army consists of N ($1 \le N \le 100$) soldiers numbered from 1 to N. PEG has sent S ($0 \le S \le N$) spies to infiltrate the cow army. Each of the spies has chosen a distinct cow soldier to abduct and steal the identity of. In other words, S of the N cow soldiers are now secretly imposters.

The commander of the cow army, Bo Vine, is a sharp leader. Recently, he has sensed something amiss with his subordinates and would like to find out if his army has been compromised. Being one of the greatest military strategists of our time, he had long planned for the day that something like this might happen. During the inception of the cow army, a secret system to identify spies was established. This system is based on the fact that all real cows have perfect pitch, and can moo at any musical frequency at will. In the past, Bo Vine had chosen a single secret musical note to share with every cow soldier on the first day of basic training. The fundamental frequency of the secret note to moo is F ($1 \le F \le 10000$) Hz, where F is an integer. When questioned, they must moo this note or be revealed as a spy.

Bo Vine has questioned all of the cows individually and gathered some data. He has recorded that the cow numbered i moos at a frequency of M_i ($F \le M_i \le 10000$) Hz, where M_i is an integer. However, upon inspecting the data, Bo Vine has noticed a big problem with his plan. He realized that he had only told the name of the secret note to the cows, and not its exact frequency. He forgot that the same musical note can be mooed in different octaves, and hence may have different frequencies. Thus, each real cow soldier could have mooed the secret note at frequency F, or at some higher octave with a frequency larger than the fundamental frequency F.

To give you a little background in music theory, pitches that are octaves above a given fundamental pitch will have frequencies 2x, 4x, 8x, and all other powers of two times the fundamental frequency. For example, if Bo Vine chose the secret note to be A at 440 Hz as the fundamental frequency, then any cows mooing at frequencies 440 Hz, 880 Hz, 1760 Hz, 3520 Hz, etc. should be deemed to be real cow soldiers. Any other frequency is indicative of an imposter! Given the fundamental frequency and the moo frequency of every cow soldier, please help Bo Vine identify which of them are actually alien imposters.

Subtasks

In test cases worth 5/10 of the points, $F = 1$ and $1 \le M_i \le 3$ for each i.

Input Format

The first line of input consists of two space-separated integers, N and F.
N lines follow, the i-th of which consists of a single integer, M_i, for $i = 1..N$.

Output Format

Output *S* lines with a single integer per line, the numbers of all of the cows that are deemed to be imposters. You should output these numbers in ascending order.

Sample Input

```
6 440
880
500
3520
441
1320
440
```

Sample Output

```
2
4
5
```

Sample Explanation

There are 6 cow soldiers. Cows numbered 2, 4, and 5 mooed at frequencies 500 Hz, 441 Hz, and 1320 Hz respectively, which are not power-of-2 multiples above the fundamental frequency of 440 Hz.

Problem J2: Redundant Formations

12 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: http://wcipeg.com/problem/wc17fj2*

After capturing and interrogating all of the alien spies from within the cow army, Bo Vine has learned of the Party of Extraterrestrial Gangsters's plans for an imminent invasion of Earth. An important realization then dawned on him — now is not the time to engage in petty quarrels with the monkeys. All of Scarberia must join forces, at least temporarily, to face an even greater evil. Knowing this, Bo Vine approached the Head-Monkey seeking a truce. After a long night of negotiation, the stubborn Head-Monkey at last admitted to the gravity of the situation. Both the cows and monkeys thus decided to unite their armies against the fearsome PEG!

Now, Bo Vine and the Head-Monkey must discuss battle formations for each of their battalions. To keep it simple, they have both decided to stick with a standard line formation, organized into platoons of cows and monkeys in sequence. In their discussion, they will denote each possible battle formation as a non-empty string *S* of up to 20 characters. Each character in the string will either be "`C`" or "`M`", denoting either a cow platoon or a monkey platoon, respectively. Of course, the big question is: what is the best possible arrangement of platoons?

The aliens of PEG are a mysterious bunch. Bo Vine and the Head-Monkey know that they should be prepared to face all kinds of battle styles. As such, it is certainly a good idea to maximize variety in the formation they choose. Just what does this mean? For a given cow-monkey formation *S*, let us define a *sub-formation* of *S* to be any pattern of one or more cow/monkey platoons which appears at least once within *S* as a contiguous subsequence. The commanders realize that any sub-formation occurring more than once in *S* is unnecessary, especially when it overlaps with another instance of itself. To be precise, a sub-formation is considered to be *redundant* if it occurs at least twice in the original formation *S*, and two different instances of it overlap with each other (in other words, they have at least one index of *S* in common).

Given a particular formation of cow-monkey platoons that Bo Vine and the Head-Monkey are considering, can you help them count the number of redundant sub-formations?

Subtask: In test cases worth 3/12 of the points, *S* contains at most 4 characters.

Input Format

The first and only line of input consists of a single string, *S*.

Output Format

Output a single line consisting of a single integer, the number of redundant sub-formations.

Sample Input	Sample Output	Sample Explanation
`CCCMMCMMCMM`	`7`	The redundant sub-formations are "`CC`", "`CMMC`", "`CMMCM`", "`CMMCMM`", "`MMCM`", "`MMCMM`", and "`MCMM`". For example, "`CC`" is redundant because it appears in *S* starting at both the first and second indices, and these two instances overlap with each other at the second index.

Problem J3: An Interspecific Army

18 Points / Time Limit: 3.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc17fs1*

With an overall battle formation decided, the cows and monkeys are now ready to fight against the incoming alien invaders! …Well, almost. You see, the commanders realized that it is not too effective to simply line up large platoons of uniformly cows or uniformly monkeys. They must work together at a more local scale, with each cow and monkey taking immediate advantage of each others' strengths and weaknesses. As such, the commanders have decided to pair up cows and monkeys to form individual combat units on the battlefield.

Between the united armies, there are N cows and N monkeys ($1 \le N \le 10^5$). The i-th cow has a combat skill level of C_i ($1 \le C_i \le 10^6$), while the i-th monkey has a combat skill level of M_i ($1 \le M_i \le 10^6$). Each cow is to be paired up with a monkey to form a combat unit, such that there are N combat units total and each of the $2N$animals is a part of exactly one unit.

A combat unit is most effective when the skill levels of its two members don't differ too greatly. More formally, we can define the *combat skill differential* of a given unit to be the absolute difference between the combat skill levels of the two animals making up that unit. For a given pairing of the $2N$ animals, let D denote the maximum combat skill differential across all N combat units. Bo Vine and the Head-Monkey would like to pair up the animals such that D ends up being as small as possible. Can you help them determine the minimum possible value of D which can be achieved?

Subtasks

In test cases worth 2/9 of the points (4/18 for Junior), $N \le 10$ and $1 \le C_i, M_i \le 2$ for each i.
In test cases worth another 2/9 of the points (4/18 for Junior), $N \le 1000$ and $1 \le C_i, M_i \le 2$ for each i.
In test cases worth another 4/9 of the points (8/18 for Junior), $N \le 1000$.

Input Format

The first line of input consists of a single integer, N.
The second line consists of N space-separated integers, $C_1, \ldots, C_N$.
The third line consists of N space-separated integers, $M_1, \ldots, M_N$.

Output Format

Output a single integer, the minimum possible value of D which can be achieved.

Sample Input

```
4
5 1 9 1
2 7 4 6
```

Sample Output

```
3
```

Sample Explanation

One possible optimal arrangement is to pair:

- cow 1 with monkey 2 (a combat skill differential of $|5 - 7| = 2$)
- cow 2 with monkey 1 (a combat skill differential of $|1 - 2| = 1$)
- cow 3 with monkey 4 (a combat skill differential of $|9 - 6| = 3$)
- cow 4 with monkey 3 (a combat skill differential of $|1 - 4| = 3$)

The maximum of these differentials is 3, and it is not possible to go any lower.

Problem J4: Cowmmunication Network

26 Points / Time Limit: 4.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc17fs2*

Great military strategists know that a battle occurs beyond just the front lines. To prepare for the the upcoming war against the Party of Extraterrestrial Gangsters, the cows and monkeys will need to have an impeccable communications network for relaying important military commands behind the scenes. To this end, chief scientist of the bovine army Guglielmoo Marcowni has developed a powerful new technology — the radio! While his radio technology is very impressive, it sometimes suffers from the issue of poor signals. To quantify the smoothness of a network, Marcowni has developed a measure of signal quality known as *Communication Compatibility*. A large Communication Compatibility score between radios suggests a great connection, a negative score suggests a very poor connection, and a score of zero suggests a so-so connection.

The cow-monkey army is made up of N ($2 \le N \le 100{,}000$) combat units, numbered from 1 to N. Each unit carries a single radio that can communicate with other radios via specific communication channels. In total, there are M ($0 \le M \le 200{,}000$) potential communication channels amongst all of the units. Implementing the i-th communication channel would allow units A_i and B_i ($1 \le A_i, B_i \le N$; $A_i \ne B_i$) to radio each other directly, with a Communication Compatibility of C_i ($-10^6 \le C_i \le 10^6$). No two communication channels connect the same unordered pair of combat units.

Bo Vine and the Head-Monkey want to create a communication network out of one or more of the M potential communication channels. The network must be constructed such that every pair of combat units is able to communicate over the network, either directly or indirectly. If a combat unit i can communicate with both combat units j and k (either directly or indirectly), then combat units j and k are also considered able to communicate with each other indirectly.

Bo Vine and the Head-Monkey want to make their network as smooth as possible. Naturally, their choice will be based on the Communication Compability scores of the channels they choose to implement. To help them quantify the overall smoothness of the network, Marcowni has defined a benchmark known as the *Cumulative Communication Compatibility* (CCC) score. The CCC score of the network is defined to be the sum of the Communication Compatibilities of all communication channels that make up the network. Please help Marcowni determine the maximum possible CCC score that a valid network connecting all N combat units could have (note that this value may not fit into a 32-bit signed integer). Unfortunately, it's possible that no such valid network may exist, in which case you should report "`Impossible`" instead.

Subtasks

In test cases worth 6/26 of the points, $N \le 10$, $M \le 10$, and $C_i < 0$ for each i.
In test cases worth another 6/26 of the points, $N \le 1000$, $M \le 2000$, and $C_i < 0$ for each i.
In test cases worth another 12/26 of the points, $N \le 1000$, $M \le 2000$.

Input Format

The first line of input consists of two space-separated integers, N and M.
M lines follow, the i-th of which consists of three space-separated integers, A_i, B_i, and C_i, for $i = 1..M$.

Output Format

Output a single line consisting of either a single integer, the maximum possible CCC score of any valid network, or the string "`Impossible`" if no valid network exists.

Sample Input 1

```
4 5
1 2 -1
2 3 -5
3 4 -3
4 1 -2
4 2 -3
```

Sample Output 1

```
-6
```

Sample Input 2

```
5 4
1 2 5
2 3 2
3 1 -1
4 5 0
```

Sample Output 2

```
Impossible
```

Sample Explanation

In the first example, one possible optimal network is by implementing the first, third, and fourth communication channels. The CCC score of such a network is $(-1) + (-3) + (-2) = -6$.

In the second example, there is no way to allow any of the first three combat units to communicate with any of the last two units, so it is impossible to build a valid network.

Note: This statement has been altered from the version presented on the live contest to eliminate uncertainty regarding the strength of test data in the original. In particular, the bound on N has been reduced from 200,000 to 200. We apologize for the issue!

Problem J5: Explosive Ordinance Disposal

34 Points / Time Limit: 4.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc17fs3*

The Party of Extraterrestrial Gangsters has begun its invasion of Earth! Vast armies of PEG soldiers have been deployed down to the surface throughout Scarberia, and the cows and monkeys have engaged them in battle.

Amidst the fighting, however, the aliens have also transported something else to the planet's surface — a bomb with devastating nuclear power! All life in Scarberia, and perhaps the rest of Earth, would surely cease if the bomb were to detonate. Fortunately, the PEG leaders are honourable enough to give their enemies a fighting chance. As such, they've set the bomb to go off after a period of three hours, and implanted a system for defusing it. They've even included an instruction manual along with it!

On the surface of the bomb, there are N ($1 \le N \le 200$) electrical terminals. There are also $N - 1$ wires running amongst the terminals, the i-th of which runs between terminals A_i and B_i ($1 \le A_i, B_i \le N$), and is either black (if $C_i = 0$), or is otherwise white (if $C_i = 1$). The wires have been arranged such that all pairs of terminals are reachable from one another by following a sequence of wires.

Bo Vine and the Head-Monkey have gotten their hands on the bomb and its accompanying instruction manual. According to the manual, the bomb will turn itself off if the following conditions are all met:

1. Each terminal i receives an electrical current with some voltage V_i, such that V_i is a positive integer.
2. For each black wire i (such that $C_i = 0$), the greatest common divisor (GCD) of V_{Ai} and V_{Bi} is equal to 1.
3. For each white wire i (such that $C_i = 1$), the GCD of V_{Ai} and V_{Bi} is greater than 1.

Bo Vine has ordered his cow engineers to prepare the necessary electrical equipment as quickly as possible. Meanwhile, the Head-Monkey has personally taken it upon herself to come up with a set of voltages $V_{1..N}$ which will successfully satisfy the conditions to defuse the bomb. However, having realized that PEG is essentially mocking them by dispatching a bomb which may be defused so easily, she's decided to get back at them by demonstrating the monkeys' superior intelligence and successfully defusing the bomb using as little voltage as possible. Help the Head-Monkey determine the minimum possible total voltage (sum of $V_{1..N}$ values) required to get the job done. Just make sure to figure it out within three hours!

Subtask

In test cases worth 6/34 of the points, $C_1 = C_2 = \ldots = C_{N-1}$.

Input Format

The first line of input consists of a single integer, N.
$N - 1$ lines follow, the i-th of which consists of three space-separated integers, A_i, B_i, and C_i, for $i = 1..(N - 1)$.

Output Format

Output a single integer, the minimum possible total voltage required to defuse the bomb.

Sample Input

```
7
4 1 1
4 5 0
7 6 1
3 6 1
1 7 0
2 4 1
```

Sample Output

```
16
```

Sample Explanation

It's optimal to send:

- a 1-volt current to terminal 5;
- 2-volt currents to terminals 1, 2, and 4; and
- 3-volt currents to terminals 3, 6, and 7.