WOBURN CHALLENGE

2017-18 Online Round 3

Friday, March 23rd, 2018

Senior Division Problems

Automated grading is available for these problems at:
wcipeg.com

For more problems from past contests, visit:
woburnchallenge.com

Problem S1: Mutual Friends

13 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: http://wcipeg.com/problem/wc173s1*

The social network Google+ has N ($2 \le N \le 6$) users, with user IDs from 1 to N. Each pair of distinct users are either friends with one another, or not.

You're given an $N \times N$ grid of values M, with $M_{i,j}$ ($0 \le M_{i,j} \le N$) being the number of mutual friends which users i and j have in common. This corresponds to the number of other users which are friends with both i and j. Note that it does not depend on whether or not i and j are friends with one another. $M_{i,j} = M_{j,i}$ for each pair of users i and j, $M_{i,i}$ is considered to be 0 for each user i.

Given the above information, you'd like to guess which users are friends with one another. If there exists a valid set of friendships which result in the given grid of mutual friend counts, then please output the number of friendships F, followed by F lines each describing one of the friendships (with 2 integers, the IDs of the two users involved in the friendship). No two friendships should involve the same unordered pair of users. If there are multiple valid sets of friendships, you may output any of them. If there are no valid sets of friendships, output "`Impossible`" instead.

Subtasks

In test cases worth 4/13 of the points, $N \le 3$.

Input Format

The first line of input consists of a single integer, N.
N lines follow, the i-th of which consists of integers, $M_{i,1..N}$, for $i = 1..N$.

Output Format

Either the string "`Impossible`", or 1 integer F followed by F lines, the i-th of which contains 2 integers describing the i-th friendship

Sample Input 1

```
5
0 0 0 1 2
0 0 2 0 0
0 2 0 0 0
1 0 0 0 1
2 0 0 1 0
```

Sample Output 1

```
5
1 2
2 5
5 3
3 1
2 4
```

Sample Input 2

```
3
0 1 1
1 0 0
1 0 0
```

Sample Output 2

```
Impossible
```

Sample Explanations

In the first case, one valid set of 5 friendships is indicated in the sample output. For this set, users 1 and 5 have exactly 2 mutual friends (users 2 and 3) as required, users 1 and 4 have exactly 1 mutual friend (user 2) as required, and so on. Note that there exist other outputs which would also be accepted. For example, the friendship between users 2 and 4 could be replaced with a friendship between users 3 and 4, the friendships could be outputted in any order, and the users in each friendship could be swapped.

In the second case, no set of friendships amongst 3 users would result in the required grid of mutual friend counts.

Problem S2: GleamingProudChickenFunRun

23 Points / Time Limit: 6.00s / Memory Limit: 128M *Submit online: http://wcipeg.com/problem/wc173s2*

You've assembled a set of N ($1 \le N \le 300{,}000$) Twitch clips from a live stream by your favourite twitch.tv streamer. A clip is a video fragment of the stream, and the i-th clip encapsulates the exclusive time interval from A_i to B_i seconds into the stream ($0 \le A_i < B_i \le 10^9$). The clips are not all guaranteed to be distinct.

In an effort to convince your friends to start watching this stream and join you in spamming its chat, you plan to show them some of the clips. You'd like to choose the smallest possible subset S of the clips which still offer a good representation of the whole stream. In particular, each of the N total clips must have some time in common with at least one clip in S. A pair of clips have some time in common with each other if there's a positive amount of time from the stream which is present in both clips – in other words, if the intersection of their exclusive time intervals is non-empty. For example, clips with time intervals (0, 5) and (4, 10) have some time in common, while clips with time intervals (0, 5) and (5, 10) do not.

Can you determine the minimum possible number of clips which S can be made up of?

Subtasks

In test cases worth 5/23 of the points, $N \le 15$.
In test cases worth another 11/23 of the points, $N \le 1000$.

Input Format

The first line of input consists of a single integer, N.
N lines follow, the i-th of which consists of two space-separated integers, A_i and B_i, for $i = 1..N$.

Output Format

Output a single integer, the minimum possible number of clips which S can be made up of.

Sample Input

```
6
11 14
14 23
5 22
12 28
2 6
22 31
```

Sample Output

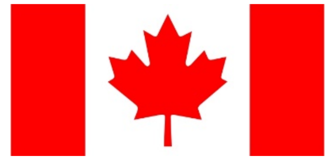
```
2
```

Sample Explanation

No single clip can be chosen such that all 5 other clips have some time in common with it. However, there are various valid sets S made up of 2 clips, such as clips 4 (12..28) and 5 (2..6).

Problem S3: Down for Maintenance

26 Points / Time Limit: 4.00s / Memory Limit: 128M *Submit online: http://wcipeg.com/problem/wc173s3*



Dave spends most of his free time on the internet, and especially loves hanging out on a hot new website. Unfortunately, in order to be able to handle the heavy traffic that it receives, the site frequently needs to be taken down for maintenance!

The site's maintenance schedule repeats every 24 hours. During each 24-hour cycle, it has one or more inactive intervals of time for which it's down. Each interval spans some positive amount of time (though it may be arbitrarily short), is less than 24 hours long, and may span from the end of one day to the start of another. All of the intervals are disjoint from one another (in other words, no point in time is inside multiple intervals, inclusive). The site is neither always up nor always down.

The schedule isn't publicly available, but Dave has noted down some observations about the site's status at various times of the day. He's made a list of N ($0 \le N \le 100{,}000$) times of day (measured in microseconds since midnight) at which he knows that the site is up, the i-th of which is U_i ($0 \le U_i < 86{,}400{,}000{,}000$). He's similarly made a list of M ($0 \le M \le 100{,}000$) times of day at which he knows that the site is down, the i-th of which is D_i ($0 \le D_i < 86{,}400{,}000{,}000$). All $N + M$ of these times are distinct.

Recently, one piece of information regarding the maintenance schedule has finally surfaced – the Government of Canada has reported that their site has exactly I ($1 \le I \le 1{,}000{,}000{,}000$) inactive intervals in each 24-hour cycle. However, Dave isn't quite sure whether the government should be trusted… As such, he'd first like to determine whether or not it's even possible for there to be exactly I inactive intervals according to his own observations.

If it is possible, he'd like to use this additional knowledge to help deduce some additional information about the maintenance schedule, in order to optimize his browsing time. He's interested in a list of K ($1 \le K \le 100{,}000$) possibly non-distinct times of day, the i-th of which is Q_i ($0 \le Q_i < 86{,}400{,}000{,}000$). For each of these times, he'd like to determine whether the site would definitely be up at that time, definitely be down, or that its status would be unknown (it might be either up or down depending on how the I inactive intervals are laid out).

Subtasks

In test cases worth 8/26 of the points, $N \le 50$, $M \le 50$, $K \le 50$, $U_i \le 100{,}000$, $D_i \le 100{,}000$, and $Q_i \le 100{,}000$.
In test cases worth another 9/26 of the points, $U_i \le 100{,}000$, $D_i \le 100{,}000$, and $Q_i \le 100{,}000$.

Input Format

The first line of input consists of a single integer, I.
The next line consists of a single integer, N.
N lines follow, the i-th of which consists of a single integer, U_i, for $i = 1..N$.
The next line consists of a single integer, M.
M lines follow, the i-th of which consists of a single integer, D_i, for $i = 1..M$.
The next line consists of a single integer, K.
K lines follow, the i-th of which consists of a single integer, Q_i, for $i = 1..K$.

Output Format

Either the string "`Impossible`" if there can't be exactly I inactive intervals, or K lines, the i-th of which is a string describing the status of time Q_i (either "`Up`", "`Down`", or "`Unknown`").

Sample Input

```
2
3
33582
9061
62057
3
78415
21592
1344
5
50000
30000
9061
66666
0
```

Sample Output

```
Up
Unknown
Up
Unknown
Down
```

Sample Explanation

Consider the second queried time of day (30,000). It's possible for the site to be up at that time, if the pair of inactive intervals are for example 31,000…35,000 and 70,000…2000. It's also possible for it to be down at that time, if the pair of inactive intervals are for example 65,000…5000 and 25,000…60,000. On the other hand, the statuses of three other queried times are guaranteed over all possible sets of two inactive intervals.

Problem S4: Relevant Results

38 Points / Time Limit: 7.00s / Memory Limit: 128M *Submit online: http://wcipeg.com/problem/wc173s4*

You own a group of N ($1 \le N \le 400{,}000$) webpages dedicated solely to the art of dynamic programming. Each page i includes links to K_i ($0 \le K_i < N$) other distinct pages, the j-th of which is page $L_{i,j}$ ($1 \le L_{i,j} \le N$). The total number of links ($\Sigma K_{1..N}$) is at most 400,000. You consider a given page b to be "accessible" from a given page a if it's possible to follow a sequence of 0 or more links starting from page a and ending at page b.

Unfortunately, your pages aren't getting as much traffic as you'd like. It's clear that the most promising way of encouraging more people to come across your pages is to somehow cause them to appear earlier in the search results when someone searches for "dynamic programming" on the internet's most popular search engine.

Each page is assigned an integral "relevancy score" for a given search query such as "dynamic programming", and pages with higher scores then appear earlier in the results for that query. Page i's initial relevancy score is R_i ($1 \le R_i \le 10^9$). Fortunately, if you donate C_i ($1 \le C_i \le 10^4$) dollars to the search engine company, you can increase page i's score by 1! You can choose to do so 0 or more times for any page.

You're not yet sure about how much money you'd like to spend on helping to make your pages "more relevant", so you'll consider a series of M ($1 \le M \le 400{,}000$) independent questions. For the i-th question, you'd like to determine the minimum amount of money you'd need to spend in order to make all N of your pages "accessible" from search results with relevancy scores no smaller than Q_i ($1 \le Q_i \le 10^9$). In other words, after you finish increasing all of the pages' scores as necessary, for each page b ($1 \le b \le N$), there must exist at least one page a such that page a's final relevancy score is greater than or equal to Q_i and page b is accessible from page a. Note that all M questions are independent, with changes to pages' scores not carrying over between them.

Subtasks

In test cases worth 9/38 of the points, $N \le 2000$, $\Sigma K_{1..N} \le 2000$, and $M = 1$.
In test cases worth another 15/38 of the points, $M = 1$.

Input Format

The first line of input consists of a single integer, N.
N lines follow, the i-th of which consists of three space-separated integers, R_i, C_i, and K_i, followed by K_i more integers $L_{i,1..K_i}$, for $i = 1..N$.
The next line consists of a single integer, M.
M lines follow, the i-th of which consists of a single integer, Q_i, for $i = 1..M$.

Output Format

Output M lines, the i-th of which should consist of a single real number, the minimum cost (in dollars) required such that all N pages are accessible from search results with relevancy scores no smaller than Q_i.

Sample Input

```
5
9 3 1 4
11 1 0
8 2 1 1
4 2 1 3
7 2 1 2
3
10
12
1
```

Sample Output

```
9
18
0
```

Sample Explanation

If page 1's relevancy score is increased to 10 (for a cost of $3) and page 5's relevancy score is also increased to 10 (for a cost of $6), then all 5 pages will be reachable from search results with relevancy scores no smaller than 10. Pages 1, 2, and 5 will have sufficiently high relevancy score themselves, and page 1 links to page 4 which then links to page 3. This total cost of $9 is optimal.