WOBURN CHALLENGE

2017-18 Online Round 4

Friday, April 20th, 2018

Senior Division Problems

Automated grading is available for these problems at:
wcipeg.com

For more problems from past contests, visit:
woburnchallenge.com

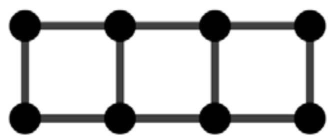
Problem S1: Wakandan Sabotage

15 Points / Time Limit: 2.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc174s1*

The Infinity War has begun! Black Panther is well aware of the importance of rallying his supporters together to help defend Earth from the evil Thanos, so he's working on unifying the Wakandan army. Unbeknownst to him, however, Loki has formed a secret alliance with Thanos and is planning on sabotaging Black Panther's efforts.

The nation of Wakanda consists of $N \times M$ cities ($2 \le N, M \le 1000$), arranged in an grid with N rows and M columns. In each of the M columns of the grid, there are $N - 1$ vertical roads connecting pairs of vertically-adjacent cities. Additionally, in the bottom-most and top-most rows only, there are $M - 1$ horizontal roads connecting pairs of horizontally-adjacent cities. As such, there are $M(N - 1) + 2(M - 1)$ roads in total throughout Wakanda. Each road is exactly 1 km long.

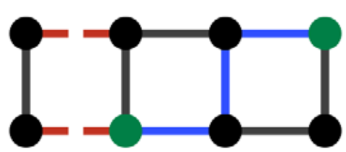
For example, if $N = 2$ and $M = 4$, the network of cities and roads looks as follows:



Loki has the explosive resources to destroy K ($0 \le K < M(N - 1) + 2(M - 1)$) roads of his choice. He'd hate for any of his explosives to go to waste, so he'll destroy exactly K roads, no fewer.

Upon realizing that some roads have been destroyed, Black Panther will still do his best to assemble the Wakandan army together. He'll ask soldiers to travel amongst various cities which are still connected by the remaining roads. Overall, the amount of time required for this mobilization will depend on a single factor – the maximum shortest distance (along undestroyed roads) between any pair of cities which are still reachable from one another at all. In other words, if $D_{i,j}$ is equal to the shortest distance between cities i and j if they're connected by roads (or is equal to 0 if they're not connected), then the value of importance is $\max\{ D_{i,j} \}$ over all pairs of cities (i, j).

For example, if Loki were to destroy the two roads indicated in red below, then the maximum shortest path between any pair of connected cities would be 3 km long (for example, between the two cities marked in green, along the roads indicated in blue).



Naturally, Loki is interested in selecting a set of K roads to destroy such that this value is maximized. Help him determine the maximum possible shortest distance between any pair of connected cities which his sabotage can result in.

Subtasks

In test cases worth 8/15 of the points, $N = 2$.

Input Format

The first and only line of input consists of three space-separated integers, N, M, and K.

Output Format

Output a single integer, the maximum achievable shortest distance between any pair of cities which are still connected (in km).

Sample Input

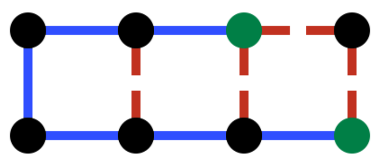
```
2 4 4
```

Sample Output

```
6
```

Sample Explanation

Loki may choose to destroy the four roads indicated in red below. The shortest path between the pair of cities marked in green would then be 6 km long (consisting of the roads indicated in blue).



Problem S2: Strange Travels

20 Points / Time Limit: 4.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc174s2*

Desperate to find more allies to join in the fight against Thanos, the Avengers have requested assistance from Doctor Strange, a powerful magician. Strange is willing to help, but he'll need some assistance of his own first. To fully apply his powers to the fight, he'll need to gather together a set of artifacts from hidden sanctums around the world!

There are N ($2 \le N \le 100{,}000$) sanctums, numbered from 1 to N, spread out all across the Earth. It would take far too long to travel amongst them by conventional means, but Doctor Strange has access to a convenient alternative – magical portals. There are M ($0 \le M \le 200{,}000$) one-way portals, with the i-th of them allowing for instantaneous travel from sanctum A_i to B_i ($1 \le A_i, B_i \le N$, $A_i \ne B_i$). No two portals connect the same pair of sanctums in the same direction.

There are K ($1 \le K \le N - 1$) artifacts which Strange requires, with the i-th of them being held in sanctum S_i ($2 \le S_i \le N$). No artifact is in sanctum 1, and no two artifacts are located in the same sanctum.

Doctor Strange is initially located in sanctum 1, also known as the Sanctum Sanctorum. He'll need to recover all K artifacts back to the Sanctum Sanctorum, one by one. In particular, for each artifact i in order, he'll need to warp through a sequence of 1 or more portals to reach sanctum S_i, collect the artifact, and then warp through a sequence of 1 or more portals to return to sanctum 1 before heading back out for the next one. Note that he may not carry multiple artifacts at a time, and must collect the K artifacts in order. Overall, he'll be required to visit the following sequence of sanctums:

$$1 \to S_1 \to 1 \to S_2 \to 1 \to \ldots \to 1 \to S_K \to 1$$

Determine the minimum number of portal warps which Doctor Strange will need to perform to achieve his goal. Unfortunately, it may instead turn out to be impossible to visit the entire required sequence of sanctums, in which case you should output –1 instead.

Subtasks

In test cases worth 6/20 of the points, $N \le 100$ and $M \le 2000$.
In test cases worth another 6/20 of the points, $N \le 2000$ and $M \le 2000$.

Input Format

The first line of input consists of two space-separated integers, N and M.
M lines follow, the i-th of which consists of two space-separated integers, A_i and B_i, for $i = 1..M$.
The next line consists of a single integer K.
K lines follow, the i-th of which consists of a single integer, S_i, for $i = 1..K$.

Output Format

Output a single integer, either the minimum number of warps required to recover all of the artifacts, or −1 if not all of them can be recovered.

Sample Input 1

```
4 6
1 2
2 3
3 1
1 3
4 3
3 4
2
4 2
```

Sample Output 1

```
7
```

Sample Input 2

```
4 5
1 2
3 1
1 3
4 3
3 4
2
4 2
```

Sample Output 2

```
-1
```

Sample Explanations

In the first case, Doctor Strange can warp through the following sequence of sanctums:

$1 \rightarrow 3 \rightarrow 4 \rightarrow 3 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 1$

In the second case, Doctor Strange would be able to recover the first artifact and then reach the second one, but he would then be unable to return to sanctum 1 with it.

Problem S3: Guardians of the Cash

28 Points / Time Limit: 4.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc174s3*

With the fate of the entire galaxy on the line, the Guardians of the Galaxy have naturally been called in to battle against Thanos as well! That being said, their leader, Star-Lord, has realized the potential for a bit of financial compensation along the way. He's successfully secured a hefty stash of solid gold coins for his team before any fighting has even begun!

While waiting for the action to finally get underway, Rocket Raccoon and Groot have decided to amuse themselves with a game involving the coins. For starters, they've drawn a grid on the ground with N rows and N columns ($1 \le N \le 1000$). Then, in each cell (i, j) (the cell in the i-th row and j-th column), they've placed a stack of $C_{i,j}$ ($1 \le C_{i,j} \le 10^9$) coins.

If you look at the resulting coin collection from the South (from beyond row 1), all you can make out is the shape of its "skyline". This Southern skyline is a sequence of N positive integers, the i-th of which is the number of coins in the tallest stack in the i-th column. Similarly, its Western skyline (when viewed from beyond column 1) is a sequence of N positive integers, the i-th of which is the number of coins in the tallest stack in the i-th row.

For example, consider the following arrangement of stack heights:

```
2 3 2
4 5 4
5 1 2
```

Its Southern skyline sequence is {5, 5, 4}, while its Western skyline sequence is {3, 5, 5}.

Rocket Raccoon now poses the following challenge to Groot: Remove 0 or more coins from the collection such that it will look exactly the same from the South as from the West (in other words, such that its Southern skyline sequence will be equal to its Western skyline sequence). Groot has a bit of trouble lifting the heavy gold coins, so he'd like to pass Rocket Raccoon's challenge while removing as few of them as possible. What's the minimum number of coins which he must remove from the stacks in total? Note that this value may not fit into a 32-bit signed integer.

Subtasks

In test cases worth 9/28 of the points, $N \le 200$ and $C_{i,j} \le 2$ for each cell (i, j).
In test cases worth another 9/28 of the points, $N \le 200$.

Input Format

The first line of input consists of a single integer, N.
N lines follow, the i-th of which consists of integers, $C_{i,1..N}$, for i = 1..N.

Output Format

Output a single integer, the minimum number of coins which Groot must remove.

Sample Input

```
3
2 3 2
4 5 4
5 1 2
```

Sample Output

```
4
```

Sample Explanation

One option is for Groot to remove one coin from the stack in cell (2, 1), one coin from (2, 3), and two coins from (3, 1), resulting in the following arrangement of stack heights:

```
2 3 2
3 5 3
3 1 2
```

Note that both its Southern and Western skyline sequences are equal to {3, 5, 3}.

Problem S4: Alpha Nerd

37 Points / Time Limit: 4.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc174s4*

After hours of exciting offscreen action (not described in this contest), the Infinity War has just about reached its conclusion, with Thanos defeated and the Avengers preparing to give a series of inspirational speeches! One important question remains – which heroes should receive the most credit for the victory? With so many superheroes crammed into a single adventure, there simply won't be enough recognition to go around!

For instance, who should receive credit for being the Avengers' alpha nerd, playing the most important tech-related role? Iron Man is the obvious choice, but Spiderman has been studying computer science for a whole few months and wants a shot at the title as well! This prospect annoys Iron Man, so he plans to give Spiderman a competitive programming problem to put him in his place. He'll even give him a nice, easy one – computing the weight of a graph's minimum spanning tree.

Iron Man will generate a complete, undirected, weighted graph with N ($2 \le N \le 300{,}000$) nodes, numbered from 1 to N. Each of its $N \times (N - 1)/2$ edges will be given a weight of either 0 or 1. Iron Man has already decided on the weights of M ($0 \le M \le N - 1$) of its edges – in particular, the edge connecting nodes A_i and B_i ($1 \le A_i, B_i \le N$) will have a weight of 0. The M fixed edges are all distinct, and there are no cycles amongst them. Each of the remaining edges in the graph will randomly receive a weight of either 0 or 1.

Spiderman is feeling confident about this challenge, having just learned about Prim's algorithm for constructing minimum spanning trees in school. Unfortunately, he's misremembered the algorithm! His version involves starting at a random node and greedily following minimum-weight edges around the graph until he's constructed a path which passes through all N nodes. In pseudocode:

1. Set c to be a random node between 1 and N, inclusive.
2. Mark node c as visited.
3. If all N nodes have been visited, terminate the algorithm.
4. Find the minimum-weight edge which connects node c to some unvisited node i (in the case of tied minimum weights, choose a random minimum-weight edge).
5. Add the edge between nodes c and i to the minimum spanning tree.
6. Set c to be equal to i.
7. Return to step 2.

This algorithm will always terminate and produce a valid spanning tree on the graph, but said spanning tree may not end up having the minimum possible weight at all. To some extent, this depends on how lucky Spiderman gets (in terms of what edge weights are randomly generated for the graph, which node is started at in Step 1 of the algorithm, and which tied minimum-weight edges are chosen in Step 4).

Assuming Spiderman gets as unlucky as possible, what could be the largest possible difference between the weight of the spanning tree returned by his algorithm, and the weight of the same graph's minimum spanning tree (which Prim's actual algorithm would return)?

Subtasks

In test cases worth 12/37 of the points, $N \leq 6$.

Input Format

The first line of input consists of two space-separated integers, N and M.
M lines follow, the i-th of which consists of two space-separated integers, A_i and B_i, for $i = 1..M$.

Output Format

Output a single integer, the maximum possible difference between the algorithm's answer and the weight of the graph's actual minimum spanning tree.

Sample Input 1

```
3 1
1 2
```

Sample Output 1

```
1
```

Sample Input 2

```
5 3
3 1
1 5
2 1
```

Sample Output 2

```
3
```

Sample Explanations

In the first case, it's possible that the edge between nodes 1 and 3 could receive a weight of 1, while the edge between nodes 2 and 3 could receive a weight of 0. It's then possible that Spiderman's algorithm could start at node 2, then randomly choose the weight-0 edge leading to node 1, and finally be forced to choose the weight-1 edge leading to node 3 before terminating. This spanning tree (2 ↔ 1 ↔ 3) has a total weight of 1, while the graph's minimum spanning tree (1 ↔ 2 ↔ 3) has a total weight of 0.

In the second case, it's possible that Spiderman's algorithm may produce a spanning tree with weight 3 greater than the minimum spanning tree.