WOBURN CHALLENGE

2018-19 Online Round 2

Friday, December 14th, 2018

Senior Division Problems

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Problem S1: Laser Grid

13 Points / Time Limit: 3.00s / Memory Limit: 32M *Submit online: http://wcipeg.com/problem/wc182s1*

The IMF (Impossible Mission Force) has dispatched their best agent, Ethan Hunt, to recover a recently stolen microchip. This microchip contains critical Canadian governmental secrets, such as the Prime Minister's favourite colour, and must be recovered before its captors have time to download its data!

Ethan has tracked the microchip down to an underground base in Saskatchewan. Upon infiltrating it, he's found himself in the middle of a gigantic, square room. When viewed from above, the room can be represented as a square on a 2D plane, with its bottom-left corner at coordinates (0, 0) and its top-right corner at coordinates (1,000,000, 1,000,000). Ethan has lowered himself down into the room, and is standing at coordinates (X_E, Y_E) $(1 \le X_E, Y_E \le 999{,}999)$.

There are N $(0 \le N \le 100{,}000)$ vertical lasers extending across the entire room, the i-th of which is a line segment from coordinates $(V_i, 0)$ to $(V_i, 1{,}000{,}000)$ $(1 \le V_i \le 999{,}999)$. There are also M $(0 \le M \le 100{,}000)$ horizontal lasers extending across the entire room, the i-th of which is a line segment from coordinates $(0, H_i)$ to $(1{,}000{,}000, H_i)$ $(1 \le H_i \le 999{,}999)$. All vertical lasers have distinct V values, all horizontal lasers have distinct H values, and no laser goes directly through Ethan's location (in other words, no V value is equal to X_E, and no H value is equal to Y_E).

Ethan was hoping to simply find the stolen microchip, but he's been greeted by a more troubling sight: there are C $(1 \le C \le 100{,}000)$ microchips strewn about the room! The i-th microchip is at coordinates (X_i, Y_i) $(1 \le X_i, Y_i \le 999{,}999)$. No two microchips are at the same location, no microchip is at Ethan's location, and no laser goes directly through any microchip's location.

One of these C microchips must be the real one, with the rest being decoys, but they all look identical! Unfortunately, Ethan will only have time to go grab at most one of them before getting out of there. To make matters even worse, Ethan may not pass through any lasers on his way to pick up the microchip of his choice, as they'd trigger an alarm. He'll need to weigh his options and choose his plan of action carefully!

For each microchip, determine whether or not Ethan would be able to reach its location from (X_E, Y_E) by following any continuous path on the 2D plane (not necessarily a straight line segment), without leaving the confines of the room and without passing through any of the $N + M$ lasers.

Subtasks

In test cases worth 5/13 of the points, each integer in the input is no greater than 10.
In test cases worth another 5/13 of the points, $N \le 2000$, $M \le 2000$, and $C \le 2000$.

Input Format

The first line of input consists of two space-separated integers, X_E and Y_E.
The next line consists of three space-separated integers, N, M, and C.
N lines follow, the i-th of which consists of a single integer, V_i, for $i = 1..N$.
M lines follow, the i-th of which consists of a single integer, H_i, for $i = 1..M$.
C lines follow, the i-th of which consists of two space-separated integers, X_i and Y_i, for $i = 1..C$.

Output Format

Output C lines with a single character per line, either "`Y`" if Ethan would be able to reach the i-th microchip, or "`N`" otherwise, for i = 1..C.

Sample Input

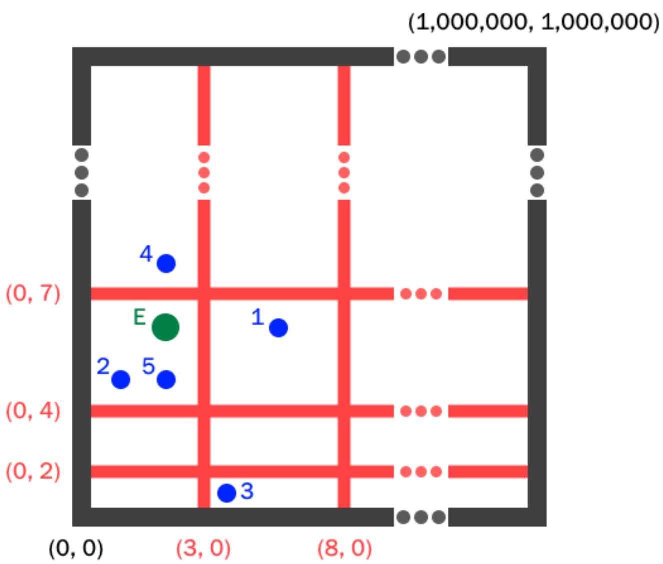
```
2 6
2 3 5
3
8
4
2
7
6 6
1 5
4 1
2 8
2 5
```

Sample Output

```
N
Y
N
N
Y
```

Sample Explanation

The room is illustrated below, with lasers indicated in red, Ethan's location in green, and the microchips in blue. Note that most of the x-coordinates and y-coordinates on the plane (from around 10 to around 999,997) have been collapsed together.



Ethan would only be able to reach the 2nd or 5th microchip.

Problem S2: Plutonium

20 Points / Time Limit: 2.00s / Memory Limit: 64M *Submit online: http://wcipeg.com/problem/wc182s2*

Ethan Hunt has traced a string of illegal plutonium trafficking down to a remote warehouse situated on a frigid island in Nunavut!

The warehouse has been operating for a period of N ($1 \le N \le 200{,}000$) days, and it had P_i ($1 \le P_i \le 10^6$) boxes of plutonium in stock at the end of the i-th day.

At the end of the first day, there was a single box ($P_1 = 1$). On each subsequent day i ($2 \le i \le N$), it's possible that a buyer came to withdraw *all* of the plutonium in the warehouse in the morning. Whether or not that occurred, one new box of plutonium then arrived at the warehouse in the afternoon. In other words, if a withdrawal occurred on day i, then $P_i = 1$, and otherwise, $P_i = P_{i-1} + 1$.

Ethan has entrusted the task of monitoring the warehouse to his friend and fellow IMF member Luther Stickell. Accurately monitoring the warehouse proved problematic, but Luther did his best to come up with a list of observations $O_{1..N}$ ($0 \le O_i \le 10^6$). If $O_i = 0$, then Luther has no idea how much plutonium the warehouse had in stock at the end of the i-th day. Otherwise, if $O_i > 0$, then Luther believes that the warehouse had exactly O_i boxes of plutonium in stock at the end of the i-th day (in other words, that $P_i = O_i$).

Ethan would now like to use Luther's observations to determine how many sellers withdrew plutonium from the warehouse. Due to gaps in Luther's observations, there may be multiple sequences of events which are consistent with them, so Ethan is interested in both the minimum and maximum possible number of withdrawals which could have taken place over the course of days 2..N. It's also possible that Luther's observations are inconsistent with any possible sequence of events.

Subtasks

In test cases worth 6/20 of the points, $N \le 20$.
In test cases worth another 8/20 of the points, $N \le 2000$.

Input Format

The first line of input consists of a single integer, N.
N lines follow, the i-th of which consists of a single integer, O_i, for i = 1..N.

Output Format

Output either two integers, the minimum and maximum possible number of withdrawals, or the single integer −1 if it's impossible for Luther's observations to be accurate

Sample Input 1

```
6
1 0 0 0 3 0
```

Sample Output 1

```
1 3
```

Sample Input 2

```
3
1 0 4
```

Sample Output 2

```
-1
```

Sample Explanation

In the first case, it's possible that plutonium was only withdrawn a single time, on the morning of the 3rd day. It's also possible for as many as 3 withdrawals to have taken place, on the mornings of the 2nd, 3rd, and 6th days.

In the second case, the warehouse could contain at most 3 boxes of plutonium at the end of the 3rd day (if no withdrawals had taken place), which is inconsistent with Luther's observed count of 4.

Problem S3: Multitasking

25 Points / Time Limit: 2.00s / Memory Limit: 128M *Submit online: http://wcipeg.com/problem/wc182s3*

Ilsa Faust, an experienced former MI6 agent, has agreed to join the IMF and work alongside Ethan Hunt!

As part of her onboarding process at the secret IMF headquarters in Ottawa, Ilsa is required to complete a training exercise involving defusing a bomb. However, she feels that simply defusing a bomb would be an insultingly easy task for her, so she's going to show off her multitasking skills and defuse N ($1 \le N \le 2{,}000$) bombs all at once!

Each bomb becomes defused as soon as two particular wires are both cut. As such, Ilsa will need to cut $2N$ wires in total to complete the exercise.

It's understandably rather difficult to keep track of that many wires, so Ilsa is going to cut the $2N$ wires in a random order. Specifically, until she's all done, she'll repeatedly select one of the remaining uncut wires at uniform random, and cut it.

Ethan was hoping to meet up with Ilsa, but he's walked in while the training session is underway, at a random point in time after she's cut some unknown number of wires x between 0 and $2N - 1$, inclusive (and so has defused between 0 and $N - 1$ bombs, inclusive). Note that all $2N$ possible values of x are equally likely before Ethan enters the room. Upon entering, he'd like to estimate how much longer Ilsa will be busy. It's too difficult to count exactly how many wires Ilsa has already cut, but Ethan counts that K ($0 \le K \le N - 1$) bombs have been defused so far. Given that fact, what's the expected number of remaining wires Ilsa has yet to cut?

Subtasks

In test cases worth 9/25 of the points, $N \le 5$.
In test cases worth another 12/25 of the points, $N \le 200$.

Input Format

The first and only line of input consists of two space-separated integers, N and K.

Output Format

Output a single real number, the expected number of remaining wires Ilsa has yet to cut. Your answer must have at most 10^{-5} absolute or relative error to be judged as correct.

Sample Input 1

```
1 0
```

Sample Output 1

```
1.5
```

Sample Input 2

```
2 0
```

Sample Output 2

```
3.125
```

Sample Input 3

```
2 1
```

Sample Output 3

```
1.25
```

Sample Explanation

In the first case, when the single bomb hasn't yet been defused, it's equally likely that Ilsa has cut either 0 or 1 wires so far (and so has either 2 or 1 wires remaining). Therefore, the expected number of wires she has yet to cut is (2 + 1)/2 = 1.5.

Problem S4: Car Convergence Chaos

42 Points / Time Limit: 4.00s / Memory Limit: 256M *Submit online: http://wcipeg.com/problem/wc182s4*

A nuclear bomb has been set up somewhere in Vancouver! Ethan Hunt and Solomon Lane have just learned of its location, and are each desperate to reach it first. If Ethan arrives on the scene first, he'll be able to shut it down, while if Solomon beats him to it, he'll surely detonate it to take the entire city down with him!

Vancouver has N ($1 \le N \le 5000$) traffic intersections lined up in a row, numbered from 1 to N in order. Ethan is currently at some intersection E, while Solomon is at a different intersection S, such that $E < S$ and $S - E$ is even. The bomb happens to be located at the intersection M which is equidistant from both E and S (such that $M = (E + S)/2$).

Ethan will drive a car through the sequence of intersections $E \to E + 1 \to \ldots \to M - 1 \to M$, while Solomon will drive through the sequence $S \to S - 1 \to \ldots \to M + 1 \to M$, until at least one of them arrives at intersection M. Though they're both in quite the hurry, disobeying traffic laws would be too reckless, so they'll patiently wait at each intersection until the lights turn green. When a car is at an intersection i, it must wait there for T_i ($1 \le T_i \le 100{,}000$) minutes before driving onwards to an adjacent intersection (either intersection $i - 1$ or $i + 1$), at which point the car will instantly move to that intersection.

For example, if $E = 3$ and $S = 7$, Ethan will spend the first T_3 minutes at intersection 3 and the next T_4 minutes at intersection 4, before reaching the bomb (intersection 5) after a total of $T_3 + T_4$ minutes. Meanwhile, Solomon will spend T_7 minutes at intersection 7 followed by T_6 minutes at intersection 6, and reach the bomb after $T_7 + T_6$ minutes. If $T_3 + T_4 = T_7 + T_6$, then both cars will reach the bomb. Otherwise, the slower car will stop driving as soon as the faster car reaches the bomb.

Meanwhile, Alan Hunley, the secretary of the IMF, will be monitoring the action closely. He'd like to keep track of how many times the identity of the agent currently closer to the bomb changes. To do so, he'll first form a Leader List, which has one entry for each minute (including both the very beginning, and the moment when a car reaches the bomb) indicating who is closer to the bomb (distance-wise) at that point in time. The distance between a car at some intersection i and the bomb (at intersection M) is $|M - i|$. Each entry in the Leader List is either "`Ethan`", "`Solomon`", or "`Neither`" (if they're both equidistant from the bomb), and the first entry (at minute 0) is always "Neither". Alan will then remove all "`Neither`" entries from the Leader List (which may cause it to become empty). Finally, he will count the number of entries in the resulting Leader List which are either different than their preceding entry, or have no preceding entry. This count is formally known as the CCCC (Closer Car Change Count).

For example, if the initial Leader List is ["`Neither`", "`Solomon`", "`Ethan`", "`Neither`", "`Ethan`", "`Ethan`", `Solomon`"], then Alan will reduce this to ["`Solomon`", "`Ethan`", "`Ethan`", "`Ethan`", "`Solomon`"], and determine its CCCC to be 3 (due to its 1st, 2nd, and 5th entries).

Due to some complex quantum time singularity science, the scenario described above will actually play out separately in one or more parallel universes, just with the initial state varying. Specifically, there is one parallel universe for each possible valid pair of starting intersections E and S (such that $E < S$ and $S - E$ is even). Considering each possible (E, S) pair independently, and computing its resulting CCCC value, what's the sum of all of these CCCC values?

Subtasks

In test cases worth 8/42 of the points, $N \le 300$ and $T_i \le 20$ for each i.
In test cases worth 18/42 of the points, $T_i \le 20$ for each i.

Input Format

The first line of input consists of a single integer, N.
The next line consists of integers, $T_{1..N}$.

Output Format

Output a single integer, the sum of all parallel universes' CCCC values.

Sample Input 1

```
4
5 6 5 3
```

Sample Output 1

```
1
```

Sample Input 2

```
7
7 3 6 2 5 2 8
```

Sample Output 2

```
9
```

Sample Explanation

In the first case, there are two possible (E, S) pairs: (1, 3) and (2, 4). The CCCC value for the first pair is 0, as the both Ethan and Solomon will be 1 intersection away from the bomb for minutes 0..4, and then 0 intersections away at minute 5, resulting in an empty Leader List. The CCCC value for the second pair is 1, as Solomon will become 0 intersections away from the bomb at minute 3 while Ethan is still 1 intersection away from it, resulting in a Leader List of ["`Solomon`"]. The sum of these CCCCs is then 0 + 1 = 1.

In the second case, there are 9 possible (E, S) pairs, with the following CCCC values:

- (1, 3): 1
- (1, 5): 1
- (1, 7): 2
- (2, 4): 1
- (2, 6): 1
- (3, 5): 1
- (3, 7): 1
- (4, 6): 0
- (5, 7): 1

The sum of these CCCCs is then 1 + 1 + 2 + 1 + 1 + 1 + 1 + 0 + 1 = 9.